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UNCERTAINTIES IN LCA • METHODOLOGY WITH CASE STUDIES

Using fuzzy numbers to propagate uncertainty in matrix-based LCI

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Abstract

Background, aim, and scope Analysis of uncertainties plays a vital role in the interpretation of life cycle assessment findings. Some of these uncertainties arise from parametric data variability in life cycle inventory analysis. For instance, the efficiencies of manufacturing processes may vary among different industrial sites or geographic regions; or, in the case of new and unproven technologies, it is possible that prospective performance levels can only be estimated. Although such data variability is usually treated using a probabilistic framework, some recent work on the use of fuzzy sets or possibility theory has appeared in the literature. The latter school of thought is based on the notion that not all data variability can be properly described in terms of frequency of occurrence. In many cases, it is necessary to model the uncertainty associated with the subjective degree of plausibility of parameter values. Fuzzy set theory is appropriate for such uncertainties. However, the computations required for handling fuzzy quantities has not been fully integrated with the formal matrix-based life cycle inventory analysis (LCI) described by Heijungs and Suh (2002).

Materials and methods This paper integrates computations with fuzzy numbers into the matrix-based LCI computational model described in the literature. The approach uses fuzzy numbers to propagate the data variability in LCI calculations, and results in fuzzy distributions of the inventory results. The approach is developed based on similarities with the fuzzy economic input–output (EIO) model proposed by Buckley (Eur J Oper Res 39:54–60, 1989).

Results The matrix-based fuzzy LCI model is illustrated using three simple case studies. The first case shows how fuzzy inventory results arise in simple systems with variability in industrial efficiency and emissions data. The second case study illustrates how the model applies for life cycle systems with co-products, and thus requires the inclusion of displaced processes. The third case study demonstrates the use of the method in the context of comparing different carbon sequestration technologies.

Discussion These simple case studies illustrate the important features of the model, including possible computational issues that can arise with larger and more complex life cycle systems.

Conclusions A fuzzy matrix-based LCI model has been proposed. The model extends the conventional matrix-based LCI model to allow for computations with parametric data variability represented as fuzzy numbers. This approach is an alternative or complementary approach to interval analysis, probabilistic or Monte Carlo techniques.

Recommendations and perspectives Potential further work in this area includes extension of the fuzzy model to EIO-LCA models and to life cycle impact assessment (LCIA); development of hybrid fuzzy-probabilistic approaches; and integration with life cycle-based optimization or decision analysis. Additional theoretical work is needed for modeling correlations of the variability of parameters using interacting or correlated fuzzy numbers, which remains an unresolved computational issue. Furthermore, integration of the fuzzy model into LCA software can also be investigated.

Keywords Data variability · Fuzzy numbers · Life cycle inventory analysis · Matrix calculations · Uncertainty

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1 Background, aim, and scope

The representation, analysis and interpretation of uncertainty are important aspects of decision-making in general and life cycle assessment (LCA) in particular (Morgan and Henrion 1990; Lloyd and Ries 2007; Reap et al. 2008a, b). Uncertainties in LCA can arise for different reasons, and at different stages of the assessment process. For example, choices made during goal and scope definition regarding system boundaries or basis for allocation in multi-product systems can affect LCA results significantly. On the other hand, in life cycle inventory analysis (LCI) uncertainties can arise from statistical variability of parameters, data gaps, or a fundamental lack of understanding of underlying physical processes. Similarly, uncertainties also arise during life cycle impact assessment (LCIA), for instance in the valuation of different environmental impacts categories. All of these factors contribute to overall uncertainty, which then affects the reliability of any findings when used for decision-making during the interpretation phase (Huijbregts 1998a, b; Bjorklund 2002; Ross et al. 2002; Lloyd and Ries 2007; Reap et al. 2008a, b). Hence, a lot of LCA research has been dedicated to developing formal approaches for representing uncertainties, performing computations with them, and interpreting their implications. The various approaches described in the literature can be broadly categorized into qualitative and quantitative techniques. Qualitative techniques include the analysis of alternative scenarios and characterization of LCI data using quality indicators (Weidema 1998).

In recent years, there has been significantly more research interest in quantitative techniques for handling uncertainties in LCA (Lloyd and Ries 2007). There are different analytic or numerical approaches used to model and propagate these uncertainties in LCA, and particularly during the LCI phase; in the latter case, the specific problem lies in determining the uncertainties of inventory results, given the corresponding uncertainties of the LCI model input parameters. The aggregate effect of uncertainties can be modeled, for instance, with probabilistic methods (Huijbregts et al. 2001; Peters 2007; Steen 2007) or interval analysis (Chevalier and Le Teno 1996). On the other hand, various analytical techniques such as sensitivity, perturbation, and key issues analysis can be used to gauge the individual effects of each uncertain parameter on the model outputs (Heijungs and Suh 2002). There has also been some work on the use of fuzzy set theory to model the variability of data and decision-making uncertainty in LCA (Geldermann et al. 2000; Tan et al. 2002, 2004; Benetto et al. 2006; Guereca et al. 2007; Seppala 2007). These works are based on the argument that fuzziness is a more appropriate way to model epistemological variability that results from different degrees of plausibility or possibility arising from human judgment (i.e., as ‘degree of belief’), while probability is more appropriate to model statistical variability related to frequency of occurrence. To illustrate the distinction, consider the hypothetical case of an LCA of prospective automotive technologies. Suppose that the goal and scope of the LCA requires fuel economy values for motor vehicles at some point in the future—for example, the year 2020. Any estimate of automobile fuel economy will have some probabilistic variability, since it will be a representative or average value of different car brands and models driven under different road conditions. This illustrates the case for which probabilistic treatment is appropriate. On the other hand, any estimate of the collective fuel economy of future motor vehicles will have to account for technological improvements relative to present-day vehicles, and such improvements can be estimated, for instance, based on predictions made by automotive experts. Such estimates are inherently subjective, and the values will be characterized by degrees of plausibility or possibility, rather than by frequency. In such cases, the variability of the data is fuzzy, rather than statistical, in nature. This paper presents a model that integrates such fuzzy variability in the formalized, matrix-based LCI model described in the literature (Heijungs and Suh 2002, 2006; Suh and Huppes 2005).

2 Methodology

The matrix-based LCI model has been described in great detail in the literature (Heijungs and Suh 2002, 2006; Suh and Huppes 2005); similarities with economic input–output (EIO) analysis and the process-based LCI model have been noted as well (Heijungs and Suh 2002; Hendrickson et al. 2006; Peters 2007). This similarity provides a basis for extending Buckley’s fuzzy EIO (1989) to the general LCI model. This methodology can be used to propagate uncertainties through the matrix-based LCI model, thereby allowing the fuzzy distributions of inventory results to be determined from the fuzzy distributions of parametric uncertainties in the model inputs. The technique is intended to determine the aggregate effect of multiple uncertain model inputs, in the same manner as Monte Carlo or interval analysis, and not the individual effect of each parameter on the LCI results.

2.1 Fuzzy numbers

Quantities with vague or imprecise values can be expressed as fuzzy numbers. The shape of a fuzzy number characterizes the degree of plausibility of different values that make up the fuzzy number. Although the distribution of fuzzy numbers can take on different shapes in principle, stylized

trapezoidal or triangular distributions are often used for simplicity (Geldermann et al. 2000; Tan et al. 2002, 2004; Benetto et al. 2006). Figure 1 shows a triangular fuzzy number with a lower bound of 5, a mode (corresponding to the most plausible value) of 10 and an upper bound of 20. This fuzzy number can be written as $(5, 10, 20)_T$, with the subscript T denoting the triangular form of the fuzzy distribution. The vertical axis shows the possibility, Π, which maps the "degree of belief" or plausibility on a scale of 0 to 1. Thus, the base of the triangle, which ranges from the lower bound to the upper bound, covers all values that are at least barely plausible or possible; this interval, (5, 20), is also known as the support. The vertex or mode of the triangle occurs at a value of 10, which can be interpreted as the most plausible value of the fuzzy number. This value is known as the core; note that, while it is a unique point in this case, the core can be an interval if the fuzzy number has a trapezoidal distribution.

It is possible to find an interval corresponding to any given degree of possibility α. For example, in Fig. 1, at α= 0.4, the resulting interval is (7, 16). This interval is known as an α-cut. Note that the support and core of a fuzzy number are also α-cuts at α values of 0 and 1, respectively. Furthermore, any fuzzy number can be decomposed into a family of α-cuts, which can then be manipulated using interval arithmetic. Hence, it is possible to implement arithmetic operations on fuzzy numbers by means of repeated interval arithmetic operations on different α-cuts (Kaufmann and Gupta 1991). The main advantage of fuzzy numbers compared to simple intervals is that they contain much more detailed information about uncertainty distributions. For instance, simple upper and lower bounds do not provide any explicit indication of central tendency and, in fact, imply that all values within the interval are equally plausible; by comparison, a fuzzy distribution can have asymmetry or skew in the uncertainties. This additional information, of course, also comes at the expense of increased computational demands. Furthermore, determining the fuzzy distributions of the model inputs will also be more difficult than simply estimating upper and lower bounds. Thus, the choice of using either simple intervals or fuzzy numbers in the model depends on the availability of sufficient data to determine distributions.

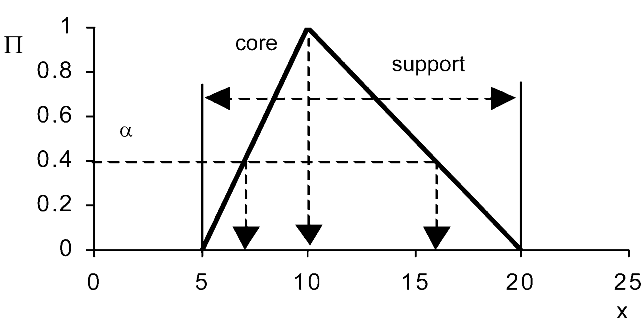

Fig. 1 Core, support, and a-cut of a triangular fuzzy number

2.2 Matrix-based LCI model

The matrix-based LCI model is discussed in great detail in Heijungs and Suh (2002, 2006) and Suh and Huppes (2005), and is described only briefly here. The flow of economic goods within the life cycle system is balanced such that the net system output is equal to the functional unit:

$$As = f \tag{1}$$

$$s = A^{-1}f \tag{2}$$

where

A	is the technology matrix
s	is the scaling vector, and
f	is the functional unit vector.

Note that Eq. 1 resembles the EIO model, with the Leontief inverse being replaced with A^{-1}. The total flow of resources and emissions of the life cycle system can be found using:

$$Bs = g \tag{3}$$

where

B	is the intervention matrix, and
g	is the inventory vector.

Combining Eqs. 2 and 3 gives the generalized LCI model:

$$g = BA^{-1}f \tag{4}$$

In this model, the convention used is that all entering flows have negative values, while all leaving flows have positive values. The columns of A and B correspond to individual processes that comprise the life cycle; the rows of A and B correspond to economic goods and environmental flows, respectively (Hendrickson et al. 2006). Finally, the relative proportions of the elements of a given process column summarize the efficiency, yield or emission factor of that process. A and B can thus be said to describe the state of technology of a life cycle system.

2.3 Fuzzy matrix-based LCI model

The basic LCI model previously described can be modified to allow for parametric variability if the elements of the component vectors and matrices are allowed to have fuzzy values. Thus, at any given α-cut, computations can be done

separately for the upper and lower bounds. Equation 2 can be modified to give:

$$s_{\mathrm{L},\alpha} = A_{\mathrm{U},\alpha}{}^{-1}f \tag{5}$$

$$s_{\mathrm{U},\alpha} = A_{\mathrm{L},\alpha}{}^{-1}f \tag{6}$$

where

$s_{U,\alpha}$ and $s_{L,\alpha}$ are the upper and lower bounds of the scaling vector, respectively, and

$A_{U,\alpha}$ and $A_{L,\alpha}$ are the upper and lower bounds of the technology matrix, respectively.

Note that the functional unit, which is specified during goal and scope definition as an exact quantity that serves as a fixed basis for all subsequent calculations, need not be fuzzy. The fuzzy inventory results for emissions can be found using:

$$g_{\mathrm{E,L},\alpha} = B_{\mathrm{E,L},\alpha}A_{\mathrm{U},\alpha}{}^{-1}f \tag{7}$$

$$g_{\mathrm{E,U},\alpha} = B_{\mathrm{E,U},\alpha}A_{\mathrm{L},\alpha}{}^{-1}f \tag{8}$$

where

$g_{E,U,\alpha}$ and $g_{E,L,\alpha}$ are the upper and lower bounds of the emissions inventory vector, respectively, and

$B_{E,U,\alpha}$ and $B_{E,L,\alpha}$ are the upper and lower bounds of the emissions intervention matrix, respectively.

Clearly, the largest emissions inventory results occur at the lowest efficiency levels, and vice-versa. Some complications arise from the convention that makes use of negative values for resource flows entering the system. The fuzzy results for resource inputs need to be calculated separately, because the mathematical upper bounds of negative fuzzy numbers actually represent smaller magnitudes of resource use, while the lower bounds correspond to the largest flows. Thus, the resource flows can be found using:

$$g_{\mathrm{R,L},\alpha} = B_{\mathrm{R,L},\alpha}A_{\mathrm{L},\alpha}{}^{-1}f \tag{9}$$

$$g_{\mathrm{R,U},\alpha} = B_{\mathrm{R,U},\alpha}A_{\mathrm{U},\alpha}{}^{-1}f \tag{10}$$

where

$g_{R,U,\alpha}$ and $g_{R,L,\alpha}$ are the upper and lower bounds of the resources inventory vector, respectively, and

$B_{R,U,\alpha}$ and $B_{R,L,\alpha}$ are the upper and lower bounds of the resources intervention matrix, respectively.

Furthermore, the conventional or non-fuzzy LCI model can be considered as a special case of the fuzzy model in which the interval upper and lower bounds have the same values. This also makes it possible to have a mix of exact and fuzzy parameters in the fuzzy LCI model.

A given fuzzy LCI model can be solved by first generating a set of upper and lower limit matrices. This is done by taking α-cuts of the original fuzzy technology and intervention matrices. A family of sub-models can then be formulated using Eqs. 7–10. Each sub-model can be solved separately to give an α-cut of the inventory vector. These α-cuts are then combined to yield the fuzzy inventory results. The computational effort needed to solve the fuzzy model depends on the number of α-cuts used. For example, if α=0, 0.1, 0.2...1, there will be a total of eleven α-cuts, and hence 22 sub-models to be solved to give a good estimate of the fuzzy distributions of the inventory results. More α-cuts will give a better resolution of the final fuzzy distributions, at the expense of additional computational effort. By comparison, simple interval arithmetic will require only two calculations, one each for the upper and lower bounds; on the other hand, Monte Carlo based methods may require repeatedly solving the same LCI model hundreds or even thousands of times. Recent research effort has also been focused on developing more efficient Monte Carlo algorithms (Peters 2007), which indicates the need to develop more computationally economical approaches to the handling of data uncertainties in LCA. In this case, direct comparison of computational demands of the probabilistic and fuzzy approaches is intended only to give some indication of the feasibility of implementation of the algorithm in practical applications, such as the possible incorporation of fuzzy numbers into commercial software. Note that the procedure described here decomposes fuzzy LCI computations into a series of repeated, deterministic LCI calculations, in a manner analogous to Monte Carlo techniques. It is therefore possible to envision software implementation of this approach in the same manner as Monte Carlo simulations.

3 Case studies

Three simple case studies are presented to illustrate the use and limitations of the model. These cases are meant to be illustrative in nature only, although they have important features that also occur in real systems. Furthermore, the fuzzy LCI model can be readily extended to much larger systems, just like the conventional matrix-based LCI model on which it is based.

It should be noted that the first two examples that follow are intended to be purely illustrative in nature, and are by necessity simplified to facilitate understanding of the meth-

Table 1 Process data for case study 1

	Process I	Process II	Total output
Product A (kg)	$(0.5, 0.6, 1)_T$	−1	100
Product B (kg)	−1	$(3, 5, 6)_T$	0
Pollutant D (kg)	$(0.05, 0.06, 0.07)_T$	$(0.9, 1, 1.1)_T$	?

odology being developed here. It is therefore assumed in these case studies that the scope of the analysis covers only one pollutant that is of interest for discussion purposes. As such, no natural resource inputs are shown in the illustrations of these systems even though they must, in reality, exist. The reader should not interpret this as a violation of fundamental material or energy balance principles; the presence of such input streams is implied, but is simply not considered in the discussion. The third case study applies fuzzy LCI to a more realistic example involving the comparison of different carbon sequestration techniques (Tan et al. 2008), and also provides a comparison of fuzzy LCI results with purely deterministic and interval-based computations.

3.1 Case 1

Table 1 shows the data for a hypothetical, two-process life cycle system with both exact and fuzzy parameters. The columns for Processes I and II denote the ratios of the flows of commodities A and B and pollutant D. The last column specifies that the functional unit is 100 kg of product A. The objective is to calculate the total emissions of pollutant D. If only the most plausible values (i.e., fuzzy core) of the parameters are used, the resulting model is no longer fuzzy and can be solved using Eq. 8. The resulting balanced system is illustrated in Fig. 2, which shows a total of 65 kg of D being released to the environment per functional unit. The system generates a net output of 100 kg of A, while all of the product B generated is consumed within the system boundaries.

The fuzzy distribution of the inventory results can be seen in Fig. 3. Note that the most plausible value is 65 kg. However, the fuzzy distribution ranges from 24–262 kg of D

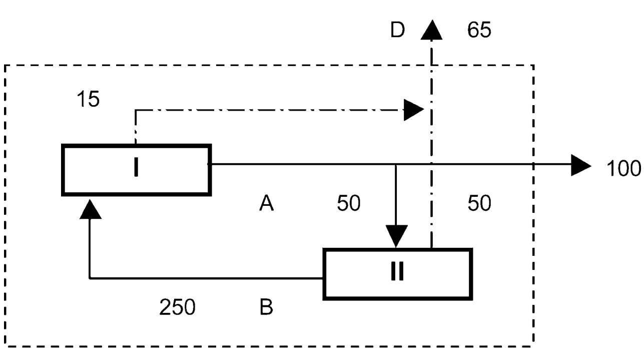

Fig. 2 Average flows in kg for case study 1

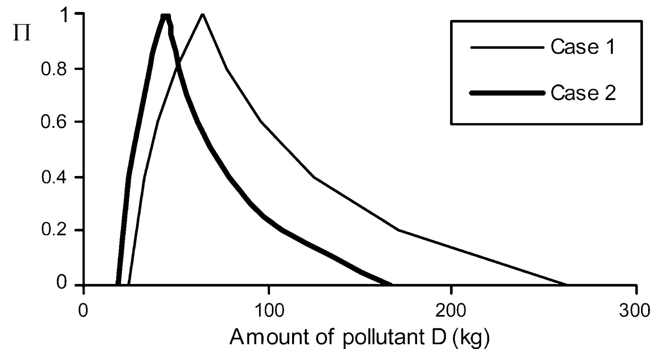

Fig. 3 Fuzzy distribution of total emissions for case studies 1 and 2

per functional unit. It is also possible to calculate fuzzy values for the internal flows of the system, although these are not shown here due to space constraints. It can also be seen that the resulting distribution is no longer triangular, but exhibits some curvature and skew so that it is convex on one side and concave on the other. This is a common feature of fuzzy arithmetic calculations (Buckley 1989; Kaufmann and Gupta 1991) and it should be noted that the resulting distributions are wider or more dispersed when compared with Monte Carlo results (Tan et al. 2002). Significantly, it is not possible to get such detailed information about the distribution using simple interval calculations; but, as previously noted, these details come only at the expense of additional computational effort. It is possible to determine, for example, the results from simple interval analysis, by using only the supports of the fuzzy numbers in Table 1. If this is done, the amount of pollutant D generated per functional unit is still 24–262 kg, which corresponds to the span of the fuzzy result shown in Fig. 3. However, it can be clearly seen that merely considering the upper and lower bounds does not give any information about the location of the most plausible or possible value, which we know from the fuzzy calculations to be 65 kg. Note that this value is significantly lower than the midpoint of the interval result, which is 143 kg. Knowledge of such details of the distribution may be essential in the subsequent impact assessment and interpretation phases of LCA. For example, techniques for ranking fuzzy numbers can be used if the LCA is intended to compare different alternatives in the presence of epistemological uncertainty (Tan et al. 2002).

Table 2 Process data for case study 2

	Process I	Process II	Process III	Total output
Product A (kg)	$(0.5, 0.6, 1)_T$	−1	0	100
Product B (kg)	−1	$(3, 5, 6)_T$	0	0
Product C (kg)	0.2	0	1	0
Pollutant D (kg)	$(0.05, 0.06, 0.07)_T$	$(0.9, 1, 1.1)_T$	$(0.2, 0.4,0.8)_T$	?

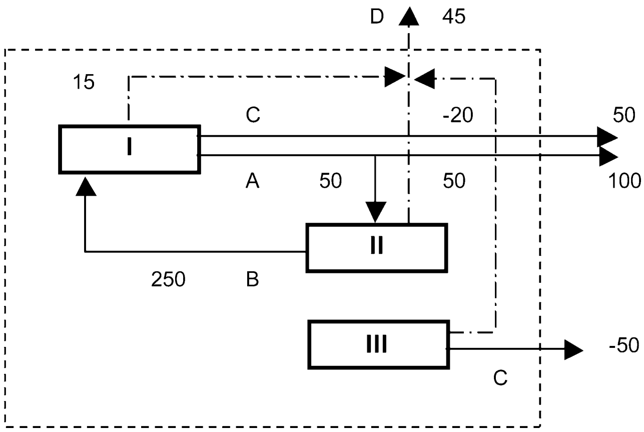

Fig. 4 Average flows in kg for case study 2

3.2 Case 2

In the matrix-based LCI model, it is possible to have negative scaling factors for displaced processes in systems with co-products. This effect is illustrated by modifying the previous case. It is now assumed that Process I also generates a second product, C. Furthermore, this product C would have otherwise been produced using an alternative industrial operation, Process III, which is now rendered unnecessary by Process I. Hence, the emissions that would have been generated by Process III can be deducted from this system as avoided emissions. The modified process data are shown in Table 2. Solving the model using the fuzzy cores leads to the system illustrated in Fig. 4, which differs from Fig. 2 only by the deduction of the flows of Process III from the total quantities. Thus, the net economic output of the system still corresponds to the specified functional unit, 100 kg of A. The most plausible value of emissions of D is only 45 kg, since the 20 kg of avoided emissions from Process III are deducted from the 65 kg generated by Processes I and II. The fuzzy distribution of the inventory result is also shown in Fig. 3; note that the values range from 19–166 kg. As in the previous case study, the support of the fuzzy number corresponds to the same result that one would get using simple interval analysis, except that the lower and upper bounds by themselves give no indication of central tendency.

3.3 Case 3

This case study involves the comparison of two alternative carbon sequestration techniques, and is based on an example from Tan et al. (2008). The goal is to determine the carbon dioxide emissions generated by a coal-fired power generation system per functional unit of output (1 MJ or electricity). The power plant is assumed to be equipped with carbon dioxide capture equipment, and that the captured emissions are then to be transferred to a CO_2 storage facility. Two options are assumed for the latter operation:

- Geological CO_2 storage, in which the CO_2 is pumped into gas-tight, underground geological formations. The operation is assumed to require a net input of electricity.
- CO_2 sequestration with enhanced coal bed methane (ECBM) recovery. In this option, the CO_2 is pumped into underground coal deposits, where it displaces methane gas. The methane is then recovered and utilized on-site to generate additional electricity. This electricity is used to meet the energy requirements to pump the CO_2 underground, and the excess power is also exported from site to the grid.

Due to space constraints, it is not possible to provide a more detailed description of the processes in this case study here. The reader is referred to the paper of Tan et al. (2008) for these details. What must be emphasized here is that many of the technologies involved in the case study are still in their infancy and, thus, prediction of their performance is subject to the same kind of epistemological uncertainties described in previous sections. Process data for the case study are given in Table 3; these are based on the same data found in Tan et al. (2008), except that uncertainty margins have been added. Inventory results for CO_2 emissions of the two competing alternatives are then shown in Table 4.

Table 3 Process data for case study 3 (adapted from Tan et al. 2008)

Process	Economic outflows	Economic inflows	Environmental outflows
Coal mining	1 kg coal	0.1 MJ power	0 kg CO_2
Power generation in coal-fired plant with CO_2 capture	$(9, 10, 12)_T$ MJ power	1 kg coal disposal service for 3 kg of CO_2	0.2 kg CO_2
Geological CO_2 storage	Disposal service for 1 kg of CO_2	$(0.4, 1, 1.8)_T$ MJ power	0 kg CO_2
CO_2 sequestration with enhanced coal bed methane recovery and integrated power generation	$(0.5, 2, 2.5)_T$ MJ power disposal service for 1 kg of CO_2	None	0.3 kg CO_2

Table 4 Fuzzy, deterministic and interval-based inventory results (in kg CO_2 per MJ of power) for case study 3

Alternative	Fuzzy LCI	Deterministic LCI	Interval LCI
Geological CO_2 storage	$(0.019, 0.029, 0.057)_T$	0.029	(0.019, 0.057)
CO_2 sequestration with enhanced coal bed methane recovery and integrated power generation	$(0.057, 0.069, 0.106)_T$	0.069	(0.057, 0.106)

A comparison of fuzzy, deterministic and interval-based LCI results can also be found in Table 4. Note that the fuzzy results provide more detailed information as compared to simple intervals. In particular, fuzzy distributions give an indication of any skew in the distributions of uncertainties, whereas simple intervals provide only upper and lower bounds, without any indication of the location of the 'best guess'. Given such results, it is likely that the midpoint of the interval will often be interpreted as the most plausible value, even if this is not necessarily the case. By comparison, the subtle features of parameter distributions can be expressed more readily using fuzzy numbers, as illustrated in these results.

4 Conclusions and perspectives

A fuzzy version of the generalized, matrix-based LCI model has been proposed. The model allows for easy computation of fuzzy inventory results by means of α-cuts for systems with ill-defined parameters in the technology and intervention matrices. Table 5 presents a SWOT (strengths, weaknesses, opportunities, and threats) analysis of the proposed methodology. There are two main advantages with this approach. The first one is that it appropriately models epistemological, non-probabilistic data variability associated with vagueness or 'degree of belief' using the concept of fuzzy possibility. While it is possible for epistemological uncertainty to be modeled using simple interval numbers, fuzzy numbers have the advantage of being able to contain much more information, which may in turn be useful for subsequent decision-making. The second advantage is that of computational efficiency, as it is possible to get the approximate fuzzy distributions of the inventory results in just a relatively small number of iterations.

The approach also can readily be extended to EIO-LCA models by incorporating Buckley's original fuzzy EIO formulation into the LCI model described here. Furthermore, it is a simple matter to extend the model to include life cycle impact assessment, as it requires only the relatively trivial step of multiplying the fuzzy inventory results by appropriate characterization, normalization and weighting factors. In practice, it is possible for the latter factors to be fuzzy as well, since subjective valuation of impact categories also involves considerable uncertainty. Furthermore, the approach can be extended to life cycle based optimization or decision analysis techniques; the computations involved are also simple enough to incorporate into LCA software.

The main computational limitation is that the model cannot account for correlations in the model parameters; further theoretical work is needed to make use of interacting (i.e., correlated) fuzzy numbers for such applications. The development of a more general LCI model that combines both fuzzy and probabilistic variability needs to be explored. Furthermore, determining the fuzzy distribution of an uncertain LCI model input parameter is, in itself, a significant challenge that needs to be addressed in future research.

Table 5 SWOT analysis of the fuzzy LCI approach

Strengths	Weaknesses	Opportunities	Threats
Appropriate to model data uncertainty resulting from vagueness or varying plausibility/possibility levels	Cannot be used to model correlated uncertainties	Extension to LCIA and EIO-LCA	Most LCA users are more familiar and comfortable with probabilistic approaches
Computationally efficient		Integration with optimization and decision analysis methods	May be incompatible with current LCI databases
		Potential for implementation in LCA software	

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